PHILOSOPHICAL TRANSACTIONS.

XII. The Bakerian Lecture.—*On the Theoretical Explanation of an apparent new Polarity in Light. By* G. B. Airy, *Esq. M.A. F.R.S., Astronomer Royal.*

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In the Report of the Seventh Meeting of the British Association*, and in that of the Eighth Meeting†, Sir David Brewster has given a short account of experiments which have led him to infer the existence of a *polarity*, in the rays of homogeneous light, having regard only to the sequence of colours in the spectrum. I am aware that Sir David Brewster has announced his intention of publishing a more elaborate paper upon these experiments. But as the leading facts of the experiments have now been communicated to the public nearly two years, and by Sir David Brewster himself, and as the accounts have been sufficiently explanatory to enable any other person to repeat the experiments, and as various persons have in consequence repeated them, there cannot, I conceive, be the most trifling impropriety in making the whole subject a matter of discussion, experimental and theoretical.

As the experimental facts observed by myself differ from those of Sir David Brewster in a degree which, as regards the experiment, is trifling (and might easily pass unobserved), but which, as regards the theory, is important; I think it necessary to give the following brief history. My first repetition of the experiment was made in October, 1839, in consequence of a friend requesting my opinion upon a proposed theoretical explanation. To that friend I immediately communicated (by letter) the experimental results at which I had arrived, with a request that he would verify them; and he did accordingly verify them, as far as the apparatus in his hands allowed. These results suggested to my mind the general train of the theoretical explanation; and the numerical calculations necessary for following it into details were immediately placed in the hands of a computer. The incessant occupations incidental to my office prevented me from even looking at the calculated numbers till the month of May, 1840, when the remarks of another friend recalled my attention to the subject.

* Transactions of the Sections, p. 12.

† Transactions of the Sections, p. 13.

I mention these matters merely for the purpose of showing distinctly that the observation of the facts was with me long anterior to the explanation, and that therefore no leaning in favour of the theory can have warped my judgment in the perception of experimental facts not wholly coinciding with Sir David Brewster's. Indeed, I should never have thought of opposing my evidence on a matter of observation to that of the Father of Modern Experimental Optics, unless the reasons for disagreement were not only, in my judgment, well-grounded, but also unmingled and independent.

The facts, then, as I have observed them, are as follows:

1. When a prismatic spectrum is viewed, *out of focus*, bands are formed by placing a piece of mica of a proper thickness (indefinite within wide limits) to cover that half of the pupil of the eye, which is on the same side as the violet end of the spectrum.

2. No bands are formed with any thickness of mica if it is placed on the same side as the red end of the spectrum.

3. When the eye is too far off to see the spectrum distinctly; then, upon passing the mica before the eye from the violet end of the spectrum, bands are seen advancing in the same direction over the spectrum.

4. When the eye is too near to the spectrum to see it distinctly; then, upon passing the mica from the violet end of the spectrum, bands are seen advancing in the opposite direction over the spectrum.

5. If the eye be so far off, and the spectrum in consequence seen so indistinctly, that the ruddy portions are nearly mingled with the blue; upon applying the mica to cover half the pupil on the side corresponding to the violet end of the spectrum, bands are seen very well defined, but narrow.

6. If the eye and mica approach to the position of distinct vision of the spectrum, the bands become somewhat broader, and near the position of distinct vision sometimes disappear. On still approaching to the spectrum, the bands reappear and become narrower, but are not, I think, seen so distinctly as when the eye is too far off for distinct vision. I know not, however, whether this may depend upon the practical difficulty of that part of the experiment.

7. Bands which are visible when the mica is on the violet side and invisible when it is on the red side, never occur when the spectrum is pure.

8. Bands are frequently visible when the spectrum is pure; but in that case they can be seen equally well, whether the mica be advanced from the violet side or from the red side.

The bands mentioned in No. 8. have been long since observed and explained by Mr. Talbot*, and of these therefore I shall make no further mention.

The general outline of the explanation of the bands seen when the spectrum is out of focus, is as follows:

* London and Edinburgh Philosophical Magazine, vol. x. p. 364.

1. By retarding the light which falls upon a portion of the pupil of the eye, bands are formed in the indistinct image produced by any one kind of homogeneous light.

2. These bands are not symmetrical with regard to the centre of the indistinct image; and the extent of asymmetry depends on the retardation.

3. Therefore if light issues from a number of luminous points of such character that the value of retardation changes gradually from one to another, the position of the bands, as measured from the centre of the indistinct image of each luminous point, will gradually change from one to another.

4. If then these luminous points be spectrally separated in one certain direction, the centres of the indistinct images will be separated in a corresponding manner, and the bands produced by all these luminous points may be made to coincide, and thereby to produce strong bands, in the confused spectrum formed by the aggregate of all the indistinct images.

5. But if the luminous points be spectrally separated in the opposite direction, the bands will be removed further from coincidence than before, and all trace of them will be lost in the confused spectrum formed by the aggregate.

6. Whether the retina be too near to the pupil, or too far from it, for distinct vision of the luminous points, the spectral separation must be such as to carry the points from which issues the most retardable light towards the side on which the retarding plate is placed.

The remainder of this paper contains the mathematical development of this explanation.

Let the wave of light, issuing from any luminous point, have, after passing the lens of the eye, the form of a spherical surface converging to the centre of the sphere. Let the radius of the spherical surface be c, and let the distance of the retina from the lens be $c + a$ (this assumption corresponds to the supposition that the luminous point is too far off for distinct vision), and let it be required to investigate the intensity of light on a point of the retina, whose distance, from the point defined by drawing a line from the luminous origin through the centre of the sphere, is b. Let x be measured from the centre of the spherical surface along that line; and let y be measured in the direction parallel to b, x and y being used to define a point on the wave surface. Then $x^2 + y^2 = c^2$. There is no necessity for introducing another co-ordinate of the wave-surface, as its effect would only be, to introduce a constant multiplier in the result.

The distance from the point x, y, to the point on the retina is

$$\sqrt{\{\overline{x+a}|^2 + \overline{y-b}|^2\}} = \sqrt{\{c^2 + 2\,a\,x - 2\,b\,y + a^2 + b^2\}}.$$

If for x we put its value $c - \frac{y^2}{2\,c}$ (as far as the second power of y), this expression becomes $\sqrt{\left\{\overline{c+a}|^2 + b^2 - 2\,b\,y - \frac{a}{c}\,y^2\right\}}$; and, expanding to the second power of

y, and putting e^2 for $\overline{c+a}|^2 + b^2$, it is

$$e - \frac{b}{e} y - \left(\frac{b^2}{2 e^3} + \frac{a}{2 c e}\right) y^2.$$

With the small values of b, of which we shall have occasion to treat, b^2 will be much less than $\frac{a e^2}{c}$. Neglecting therefore the first multiplier of y^2, and putting ϱ for the distance from the point x, y to the point on the retina,

$$\begin{aligned} \varrho &= e - \frac{b}{e} y - \frac{a}{2 c e} y^2 \\ &= e + \frac{c b^2}{2 e a} - \frac{a}{2 c e}\left(y + \frac{c b}{a}\right)^2 \\ &= f - \frac{a}{2 c e}\left(y + \frac{c b}{a}\right)^2, \end{aligned}$$

where $f = e + \frac{c b^2}{2 e a}$.

If now we suppose (as in diffractions generally) that every part of the great wave, after leaving the lens, is simultaneously the origin of a small wave diverging in all directions, at least through a large angular extent, we may represent the disturbance of ether on the point of the retina, produced by the small portion δy of the large wave, by

$$\delta y \times \sin \frac{2\pi}{\lambda}(v t - \varrho)$$

or

$$\delta y \times \sin \frac{2\pi}{\lambda}\left\{v t - f + \frac{a}{2 c e}\left(y + \frac{c b}{a}\right)^2\right\}$$

or

$$\sin \frac{2\pi}{\lambda}(v t - f) \,.\, \delta y \cos \frac{2\pi}{\lambda} \cdot \frac{a}{2 c e} \cdot \left(y + \frac{c b}{a}\right)^2$$

$$+ \cos \frac{2\pi}{\lambda}(v t - f) \,.\, \delta y \sin \frac{2\pi}{\lambda} \cdot \frac{a}{2 c e} \cdot \left(y + \frac{c b}{a}\right)^2;$$

and consequently the disturbance of ether on the point of the retina produced by the whole wave will be

$$\sin \frac{2\pi}{\lambda}(v t - f) \int_y \cos \frac{\pi a}{\lambda c e}\left(y + \frac{c b}{a}\right)^2 + \cos \frac{2\pi}{\lambda}(v t - f) \int_y \sin \frac{\pi a}{\lambda c e}\left(y + \frac{c b}{a}\right)^2.$$

The limits of y in the integral must be the limiting values which determine the extent of the great wave where it leaves the lens under the circumstances assumed, that is, as far as there is no additional cause of retardation of the wave.

But if, by the interposition of any refracting substance with parallel boundaries, a portion of the wave be retarded by the phase R (expressed as an angle), then the expression to be integrated will be

$$\int_y \sin\left(\frac{2\pi}{\lambda}\left\{v t - f + \frac{a}{2 c e}\left(y + \frac{c b}{a}\right)^2\right\} - \mathrm{R}\right)$$

or

$$\sin \frac{2\pi}{\lambda}(vt-f)\int_y \cos\left\{\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2-\mathrm{R}\right\}$$
$$+\cos \frac{2\pi}{\lambda}(vt-f)\int_y \sin\left\{\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2-\mathrm{R}\right\}$$

or

$$\sin \frac{2\pi}{\lambda}(vt-f)\left\{\cos \mathrm{R}\,.\int_y \cos\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2+\sin \mathrm{R}\,.\int_y \sin\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2\right\}$$
$$+\cos \frac{2\pi}{\lambda}(vt-f)\left\{\cos \mathrm{R}\,.\int_y \sin\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2-\sin \mathrm{R}\,.\int_y \cos\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2\right\},$$

where the limits of y in the integral must be the values which determine the extent of that portion of the great wave which is affected by the interposition of the refracting substance.

Suppose now that a plate of mica or other refracting substance is so placed as to cover a portion of the pupil on that side on which y is considered positive; and suppose that its limits, measured in the same way, are from $y = + g$ to y indefinitely great: suppose also that the limits of the pupil are from $y = - h$ to $y = + h$; then the integral which is independent of R must be taken from $y = - h$ to $y = + g$, and that which depends on R must be taken from $y = + g$ to $y = + h$, and the integrals must be added to obtain the whole disturbance of ether at the point of the retina.

Our expressions then depend entirely on the two integrals $\int_y \cos\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2$ and $\int_y \sin\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2$. For estimating these, we shall refer to FRESNEL's invaluable table of $\int_s \cos\frac{\pi}{2}s^2$ and $\int_s \sin\frac{\pi}{2}s^2$*. If $s = \sqrt{\frac{2a}{\lambda c e}}\cdot\left(y+\frac{cb}{a}\right)$, it is easily seen that $\int_y \cos\frac{\pi a}{\lambda c e}\left(y+\frac{cb}{a}\right)^2 = \sqrt{\frac{\lambda c e}{2a}}.\int_s \cos\frac{\pi}{2}s^2$, the limiting values of s being found from those of y by the equation $s = \sqrt{\frac{2a}{\lambda c e}}\cdot\left(y+\frac{cb}{a}\right)$. Omitting the constant multiplier $\sqrt{\frac{\lambda c e}{2a}}$ (remarking only that, as it becomes infinite when $a = 0$, and impossible when a is negative, different investigations must then be used); and putting C (s) for $\int_s \cos\frac{\pi}{2}s^2$ and S (s) for $\int_s \sin\frac{\pi}{2}s^2$; and remarking that, as FRESNEL's integrals vanish when $s = 0$, and as the differential coefficients have the same values with the same sign for negative and positive values of s, the integrals must have equal values but with opposite signs for negative and positive values of s, or that $\mathrm{C}(-s) = -\mathrm{C}(s)$, $\mathrm{S}(-s) = -\mathrm{S}(s)$; we find that the integral, in the case before us, may be represented as follows:

$$\sin\frac{2\pi}{\lambda}(vt-f)\left\{\mathrm{C}\left(\sqrt{\frac{2a}{\lambda c e}}\cdot h-\frac{cb}{a}\right)+\mathrm{C}\left(\sqrt{\frac{2a}{\lambda c e}}\cdot g+\frac{cb}{a}\right)\right\}$$

* Mémoire sur la Diffraction, Mém. de l'Institut, 1821 and 1822.

$$+ \cos\frac{2\pi}{\lambda}(vt-f)\left\{\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h-\frac{cb}{a}}\right)+\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}$$

$$+ \sin\frac{2\pi}{\lambda}(vt-f).\cos\mathrm{R}\left\{\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h+\frac{cb}{a}}\right)-\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}$$

$$+ \sin\frac{2\pi}{\lambda}(vt-f).\sin\mathrm{R}\left\{\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h+\frac{cb}{a}}\right)-\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}$$

$$+ \cos\frac{2\pi}{\lambda}(vt-f).\cos\mathrm{R}\left\{\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h+\frac{cb}{a}}\right)-\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}$$

$$- \cos\frac{2\pi}{\lambda}(vt-f).\sin\mathrm{R}\left\{\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h+\frac{cb}{a}}\right)-\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}.$$

The expression in this general form is rather troublesome. If, however, we suppose a not to be exceedingly small, so that $\sqrt{\frac{2a}{\lambda ce}}\cdot h$ is equal to several units, and if we remark that the values of C (s) and S (s) approach rapidly to the limit $\frac{1}{2}$, it will be seen that (for all values of g which are not very nearly equal to h) we shall commit no sensible error in putting $\frac{1}{2}$ for $\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h+\frac{cb}{a}}\right)$, $\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h-\frac{cb}{a}}\right)$, $\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h+\frac{cb}{a}}\right)$, $\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{h-\frac{cb}{a}}\right)$.

Making this substitution, our integral may be put into this form:

$$\sin\frac{2\pi}{\lambda}(vt-f)\times\left[\frac{1}{2}+\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)+\cos\mathrm{R}\left\{\frac{1}{2}-\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}\right.$$
$$\left.+\sin\mathrm{R}\left\{\frac{1}{2}-\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}\right]$$

$$+\cos\frac{2\pi}{\lambda}(vt-f)\times\left[\frac{1}{2}+\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right.$$
$$\left.+\cos\mathrm{R}\left\{\frac{1}{2}-\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}-\sin\mathrm{R}\left\{\frac{1}{2}-\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}\right].$$

The intensity of light being estimated, as is usual in the theory of undulations, by the sum of the squares of the coefficients of $\sin\frac{2\pi}{\lambda}(vt-f)$ and $\cos\frac{2\pi}{\lambda}(vt-f)$, we find the following expression for the intensity of light on the point of the retina:

$$1+2\left\{\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}^2+2\left\{\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}^2$$

$$+\cos\mathrm{R}\times\left[1-2\left\{\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}^2-2\left\{\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right\}^2\right]$$

$$+\sin\mathrm{R}\times\left[2\,\mathrm{C}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)-2\,\mathrm{S}\left(\sqrt{\frac{2a}{\lambda ce}}\cdot\overline{g+\frac{cb}{a}}\right)\right].$$

Before entering into the detailed consideration of the numerical value of this expression, we will make a similar investigation for the supposition that the luminous point is too near for distinct vision.

Let the distance of the retina from the lens be $c - a'$. The distance ϱ, of a point of the wave whose ordinate is y, from the point of the retina whose ordinate is b, is

$$\varrho = \sqrt{\{\overline{x - a'}|^2 + \overline{y - b}|^2\}} = \sqrt{\{c^2 - 2\,a'\,x - 2\,b\,y + a'^2 + b^2\}} =$$

$$\sqrt{\left\{\overline{c - a'}|^2 + b^2 - 2\,b\,y + \frac{a'}{c}\,y^2\right\}}$$; and putting $e^2 = \overline{c - a'}|^2 + b^2$, and expanding as before, this becomes

$$\varrho = e - \frac{b}{e}\,y + \frac{a'}{2\,c\,e}\,y^2$$

$$= f + \frac{a'}{2\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2,$$

where $f = e - \dfrac{c\,b^2}{2\,e\,a'}$.

The integral which expresses the disturbance of ether at the point of the retina is now

$$\sin\frac{2\,\pi}{\lambda}\,(v\,t - f)\int_y \cos\frac{\pi\,a'}{\lambda\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2 - \cos\frac{2\,\pi}{\lambda}\,(v\,t - f)\int_y \sin\frac{\pi\,a'}{\lambda\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2$$

to be taken from $y = -h$ to $y = +g$. And

$$\sin\frac{2\,\pi}{\lambda}(v\,t - f)\left\{\cos \mathrm{R}\cdot\int_y \cos\frac{\pi\,a'}{\lambda\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2 - \sin \mathrm{R}\int_y \sin\frac{\pi\,a'}{\lambda\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2\right\}$$

$$- \cos\frac{2\,\pi}{\lambda}\,(v\,t - f)\left\{\cos \mathrm{R}\,.\int_y \sin\frac{\pi\,a'}{\lambda\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2 + \sin \mathrm{R}\int_y \cos\frac{\pi\,a'}{\lambda\,c\,e}\left(y - \frac{c\,b}{a'}\right)^2\right\}$$

to be taken from $y = +g$ to $y = +h$.

Proceeding exactly as before, omitting the constant multiplier $\sqrt{\dfrac{\lambda\,c\,e}{2\,a'}}$, supposing h to be large in the integral, and estimating the intensity of light on the point of the retina as before, we obtain this expression :

$$1 + 2\left\{\mathrm{C}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{g - \frac{c\,b}{a'}}\right)\right\}^2 + 2\left\{\mathrm{S}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{g - \frac{c\,b}{a'}}\right)\right\}^2$$

$$+ \cos \mathrm{R} \times \left[1 - 2\left\{\mathrm{C}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{g - \frac{c\,b}{a'}}\right)\right\}^2 - 2\left\{\mathrm{S}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{g - \frac{c\,b}{a'}}\right)\right\}^2\right]$$

$$+ \sin \mathrm{R} \times \left[2\,\mathrm{S}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{g - \frac{c\,b}{a'}}\right) - 2\,\mathrm{C}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{g - \frac{c\,b}{a'}}\right)\right]$$

or

$$1 + 2\left\{\mathrm{C}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{\frac{c\,b}{a'} - g}\right)\right\}^2 + 2\left\{\mathrm{S}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{\frac{c\,b}{a'} - g}\right)\right\}^2$$

$$+ \cos \mathrm{R} \times \left[1 - 2\left\{\mathrm{C}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{\frac{c\,b}{a'} - g}\right)\right\}^2 - 2\left\{\mathrm{S}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{\frac{c\,b}{a'} - g}\right)\right\}^2\right]$$

$$+ \sin \mathrm{R} \times \left[2\,\mathrm{C}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{\frac{c\,b}{a'} - g}\right) - 2\,\mathrm{S}\left(\sqrt{\frac{2\,a'}{\lambda\,c\,e}}\cdot\overline{\frac{c\,b}{a'} - g}\right)\right].$$

This expression is exactly similar to that obtained for the case when the luminous point is too distant for distinct vision, in the following particulars, which we shall find to be very important. First, the general form is the same. Secondly, the argument of the functions C and S has the quantity b with the positive sign in both expressions; and therefore, in both expressions, the argument increases as b increases. We shall also find the following remark to be not without importance; that, in the arguments of the functions in the two expressions, the quantity g enters with different signs.

Our expressions now depend entirely upon the three functions,

$$1 + 2\,(C(s))^2 + 2\,(S(s))^2,$$
$$1 - 2\,(C(s))^2 - 2\,(S(s))^2,$$
$$2\,C\,(s) - 2\,S(s),$$

which we shall call A(s), D(s), and E(s). Adopting Fresnel's values of C(s) and S(s), I have computed the following values of A(s), D(s), and E(s).

s.	A(s).	D(s).	E(s).	s.	A(s).	D(s).	E(s).
0·0	+1·000	+1·000	0·000	2·9	+1·969	+0·031	+0·306
0·1	+1·020	+0·980	+0·199	3·0	+2·227	−0·227	+0·220
0·2	+1·080	+0·920	+0·391	3·1	+2·308	−0·308	−0·039
0·3	+1·180	+0·820	+0·571	3·2	+2·139	−0·139	−0·253
0·4	+1·318	+0·682	+0·728	3·3	+1·869	+0·131	−0·226
0·5	+1·493	+0·507	+0·856	3·4	+1·754	+0·246	+0·019
0·6	+1·700	+0·300	+0·942	3·5	+1·912	+0·088	+0·236
0·7	+1·929	+0·071	+0·976	3·6	+2·176	−0·176	+0·193
0·8	+2·169	−0·169	+0·949	3·7	+2·249	−0·249	−0·064
0·9	+2·401	−0·401	+0·852	3·8	+2·042	−0·042	−0·234
1·0	+2·601	−0·601	+0·685	3·9	+1·809	+0·192	−0·105
1·1	+2·743	−0·743	+0·457	4·0	+1·850	+0·150	+0·157
1·2	+2·802	−0·802	+0·186	4·1	+2·111	−0·111	+0·197
1·3	+2·758	−0·758	−0·093	4·2	+2·221	−0·221	−0·042
1·4	+2·609	−0·609	−0·339	4·3	+2·018	−0·018	−0·208
1·5	+2·371	−0·371	−0·502	4·4	+1·812	+0·189	−0·047
1·6	+2·084	−0·084	−0·545	4·5	+1·930	+0·070	+0·184
1·7	+1·814	+0·186	−0·449	4·6	+2·176	−0·176	+0·103
1·8	+1·630	+0·370	−0·233	4·7	+2·126	−0·126	−0·150
1·9	+1·591	+0·409	+0·043	4·8	+1·870	+0·130	−0·125
2·0	+1·713	+0·287	+0·291	4·9	+1·879	+0·122	+0·131
2·1	+1·957	+0·043	+0·416	5·0	+2·133	−0·133	+0·130
2·2	+2·225	−0·225	+0·363	5·1	+2·132	−0·132	−0·124
2·3	+2·398	−0·398	+0·149	5·2	+1·879	+0·121	−0·115
2·4	+2·385	−0·385	−0·128	5·3	+1·903	+0·097	+0·135
2·5	+2·186	−0·186	−0·322	5·4	+2·149	−0·149	+0·087
2·6	+1·908	+0·092	−0·321	5·5	+2·070	−0·070	−0·150
2·7	+1·719	+0·281	−0·120				
2·8	+1·744	+0·256	+0·153	∞	+2·000	0·000	0·000

For negative values of s, the numerical values of A(s), D(s), and E(s) are the same as for the equal positive values of s: the signs of A(s) and D(s) are the same for the positive and negative values of s, but the signs of E (s) are different.

The first remark which presents itself, in looking at this table, is, that the numbers D (s) and E (s) increase and diminish, and then change signs and increase and diminish, with increasing values of s; that the maximum of one always coincides nearly with the vanishing point of the other; and that the maximum values are nearly equal in both series (regard being had to the general diminution of the numbers as s increases); in fact, that these numbers may be represented as the cosine and sine of the same arc, the radius being slowly variable. To prove this, let us make D (s) $=$ G (s) . cos ϕ (s), E (s) $=$ G (s) sin ϕ (s), and investigate the values of G (s) and ϕ (s) for every value of s in the Table. They are easily found by the formulæ

$$\mathrm{G}(s) = \sqrt{\left\{\overline{\mathrm{D}(s)}|^2 + \overline{\mathrm{E}(s)}|^2\right\}}, \quad \tan . \phi(s) = \frac{\mathrm{E}(s)}{\mathrm{D}(s)}.$$

In this manner the following Table is formed; in which, for more distinct appreciation of the progress of ϕ (s), the differences of ϕ (s) are set down.

s.	G (s).	ϕ (s).	Difference.	s.	G (s).	ϕ (s).	Difference.
		° ′	° ′			° ′	° ′
0·0	1·000	0 0	11 28	2·9	0·307	720° + 84 14	51 29
0·1	1·000	11 28	11 37	3·0	·316	135 43	51 27
0·2	1·000	23 5	11 47	3·1	·311	187 10	54 1
0·3	0·999	34 52	12 0	3·2	·288	241 11	58 56
0·4	·997	46 52	12 29	3·3	·263	300 7	64 15
0·5	·993	59 21	12 54	3·4	·249	1080° + 4 22	65 8
0·6	·988	72 15	13 36	3·5	·252	69 30	63 0
0·7	·979	85 51	14 17	3·6	·261	132 30	62 1
0·8	·963	100 8	15 7	3·7	·256	194 31	65 21
0·9	·943	115 15	16 0	3·8	·235	259 52	71 24
1·0	·909	131 15	17 11	3·9	·218	331 16	75 0
1·1	·869	148 26	18 27	4·0	·217	1440° + 46 16	72 59
1·2	·822	166 53	20 9	4·1	·226	119 15	71 24
1·3	·762	187 2	22 5	4·2	·225	190 39	74 29
1·4	·696	209 7	24 24	4·3	·209	265 8	80 51
1·5	·622	233 31	27 41	4·4	·195	345 59	83 15
1·6	·552	261 12	31 19	4·5	·198	1800° + 69 14	80 23
1·7	·490	292 31	35 14	4·6	·204	149 37	80 23
1·8	·441	327 45	38 18	4·7	·195	230 0	86 15
1·9	·413	360° + 6 3	39 12	4·8	·180	316 15	90 58
2·0	·424	45 15	38 49	4·9	·180	2160° + 47 13	88 25
2·1	·419	84 4	37 43	5·0	·186	135 38	87 39
2·2	·428	121 47	37 43	5·1	·180	223 17	93 15
2·3	·424	159 30	38 53	5·2	·169	316 32	97 53
2·4	·404	198 23	41 34	5·3	·169	2520° + 54 25	95 8
2·5	·372	239 57	46 2	5·4	·173	149 33	95 22
2·6	·335	285 59	50 54	5·5	·166	244 55	
2·7	·309	336 53	53 59				
2·8	0·300	720° + 30 52	53 22	∞	·000		

For negative values of s, the numerical values of G (s) and ϕ (s) are the same as for equal positive values of s; the sign of G (s) is always positive, but the sign of ϕ (s) is the same as that of s.

The uniformity in the progress of the values, both of G (s) and of φ (s), is very remarkable; and seems to render it probable that, although the functions D (s) and E (s) have been formed in such different ways from C (s) and S (s), and although C (s) and S (s) are formed in a manner which scarcely permits a simple relation between them, yet some simple expression for G (s) and φ (s) must exist. This presumption becomes stronger, when we examine the values of s which correspond to the quadrantal values of φ (s). If we make φ (s) successively equal to 0, 90°, 180°, 270°, &c., and take the corresponding values of s, and the squares of those values, we form the following series.

Quadrant of ϕ (s).	Value of s.	Value of s^2.	Quadrant of ϕ (s).	Value of s.	Value of s^2.
0	0·00	0·00	16	3·94	15·5
1	0·73	0·53	17	4·06	16·5
2	1·27	1·61	18	4·18	17·5
3	1·63	2·56	19	4·31	18·6
4	1·88	3·53	20	4·42	19·6
5	2·12	4·50	21	4·53	20·6
6	2·36	5·60	22	4·64	21·5
7	2·57	6·60	23	4·75	22·5
8	2·74	7·50	24	4·85	23·5
9	2·91	8·49	25	4·95	24·5
10	3·09	9·5	26	5·06	25·5
11	3·25	10·6	27	5·15	26·6
12	3·40	11·6	28	5·24	27·5
13	3·53	12·5	29	5·34	28·5
14	3·67	13·5	30	5·44	29·6
15	3·81	14·5			

The values of s^2 corresponding to the successive quadrants of φ (s) increase pretty uniformly every time by 1, except in the first instance, where the increase is 0·5. This interruption of the law is not without analogues in Physical Optics. I may mention the expression for the intensity of light produced by a grating before a lens, namely $\left(\frac{\sin n\,\theta}{\sin\theta}\right)^2$, of which the first and greatest maximum occurs when $\theta = 0$, and the succeeding maxima occur nearly when $\theta = \frac{\pi}{2\,n}, \frac{3\,\pi}{2\,n}, \frac{5\,\pi}{2\,n}$, &c.

The small irregularities in the progress of these various numbers may probably arise, as I think, from irregularities in Fresnel's calculation of the original integrals. Several years ago I verified a portion of Fresnel's table; and though the agreement of my numbers with Fresnel's was sufficiently close to show that the numbers might be used with perfect safety, yet there were some small discordances which seemed to indicate that a complete recalculation might be useful.

Adopting now for $1 - 2\,(\mathrm{C}\,(s))^2 - 2\,(\mathrm{S}\,(s))^2$ and $2\,\mathrm{C}\,(s) - 2\,\mathrm{S}\,(s)$, the expressions G ($s$) cos φ (s) and G (s) sin φ (s), and observing that $1 + 2\,(\mathrm{C}\,(s))^2 + 2\,(\mathrm{S}\,(s))^2 = 2 - \mathrm{G}\,(s)\cos\varphi\,(s)$, the expression for the intensity of light, on the point of the re-

Figure 1.

Curve representing the values of $G(s).\ \cos\varphi(s)$

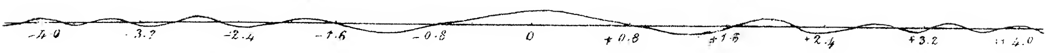


Figure 2.

Curve representing the values of $G(s).\cos\{\varphi(s)-90^\circ\}$

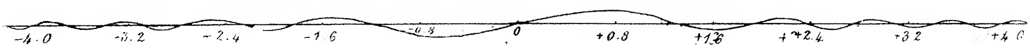
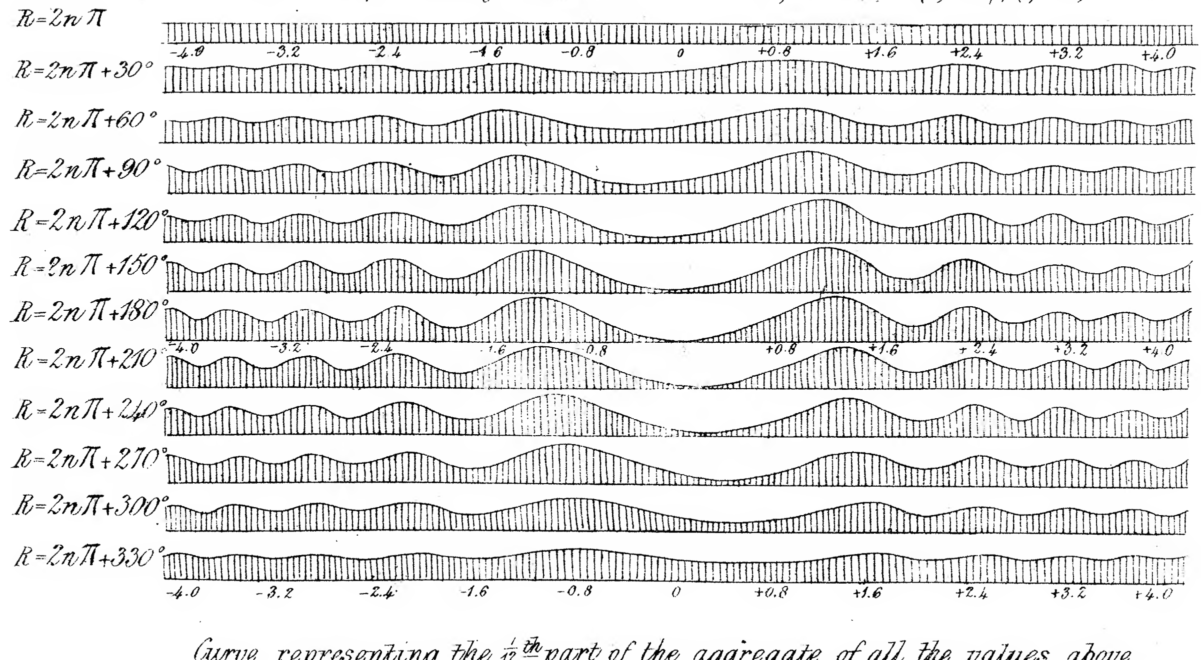


Figure 3

Curves representing the values of $2-G(s)\cos\varphi(s)+G(s)\cos\{\varphi(s)-R\}$



Curve representing the $\frac{1}{12}$th part of the aggregate of all the values above

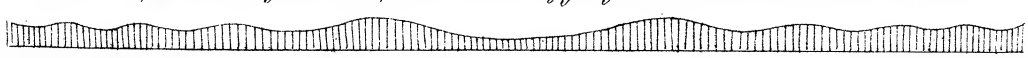
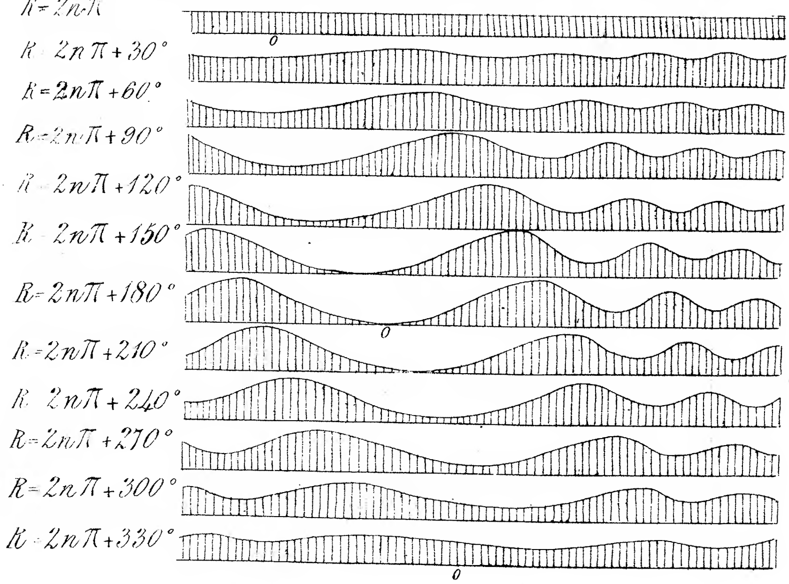
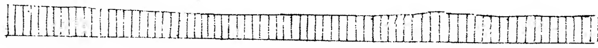


Figure 4

Curves representing the values of $2-G(s).\cos\varphi(s).+G(s).\cos\{\varphi(s)-R\}$. supposing the origin of s shifted 0.15 for each change of 30° in R, in the direction corresponding to the case of the red end of the spectrum on the same side as the retarding plate.



Curve representing $\frac{1}{12}$ part of the aggregate of the values above.



Curve representing $\frac{1}{24}$ part of the aggregate, supposing the number of curves increased to 24.

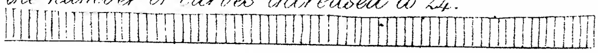
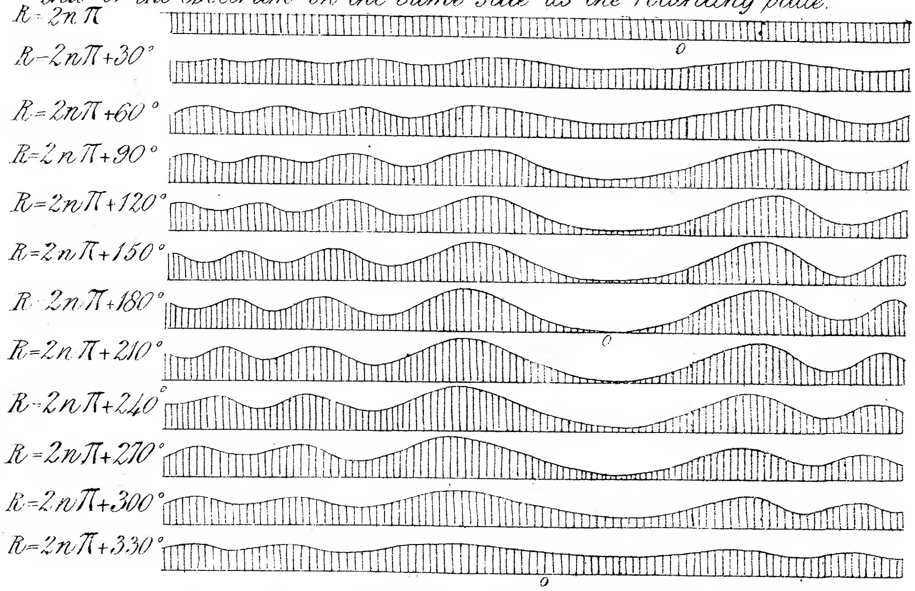
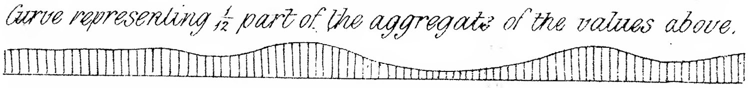


Figure 5.

Curves representing the values, supposing the origin of s shifted 0.1 for each change of 30° in R, in the direction corresponding to the case of the violet end of the spectrum on the same side as the retarding plate.

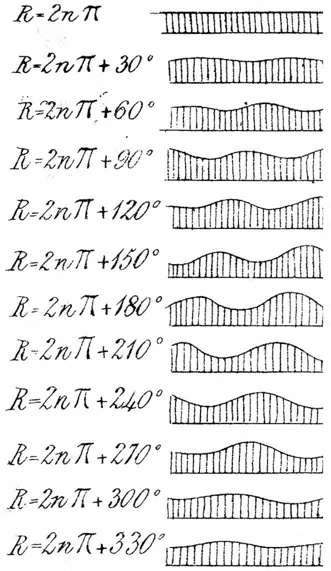


Curve representing $\frac{1}{12}$ part of the aggregate of the values above.



Curve representing $\frac{1}{24}$ part of the aggregate, supposing the number

Curves representin
of s shifted 0.15
direction correspo
the spectrum on th



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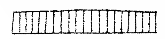

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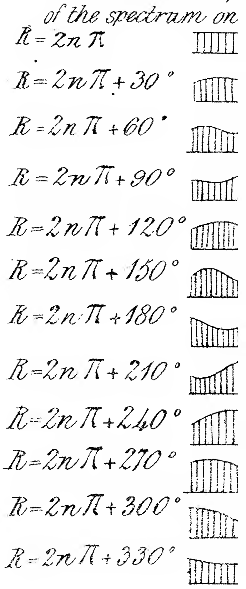
the number o

Curves representi
of s shifted 0.2
direction corresp
of the spectrum on



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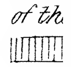
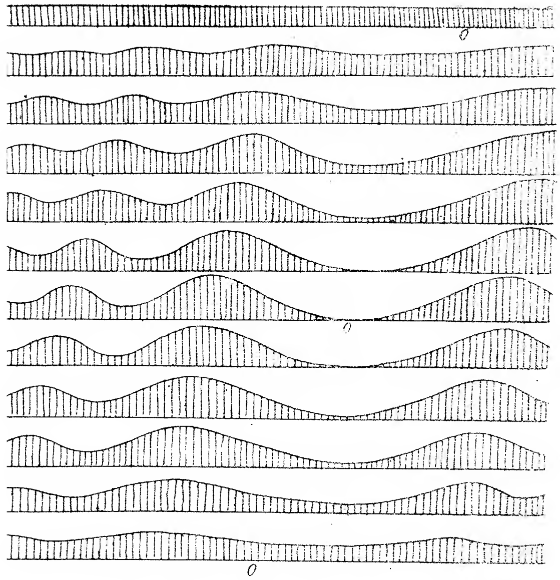
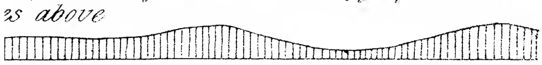


Figure 6.

resenting the values, supposing the origin d 0.15 for each change of 30° in R. in the corresponding to the case of the violet end of m on the same side as the retarding plate.



representing $\frac{1}{12}$ part of the aggregate of the es above



representing $\frac{1}{24}$ part of the aggregate, supposing umber of curves increased to 24.

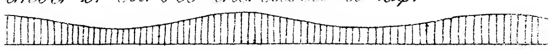
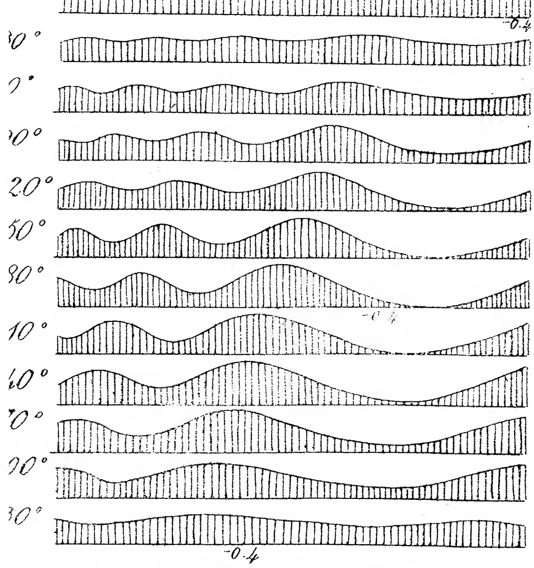
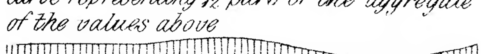


Figure 7.

epresenting the values, supposing the origin ted 0.2 for each change of 30° in R, in the corresponding to the case of the violet end trum on the same side as the retarding plate.

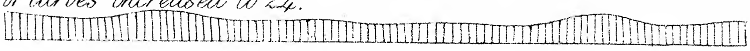


Curve representing $\frac{1}{12}$ part of the aggregate of the values above



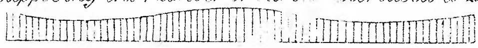
Curve representing the $\frac{1}{12}$th part of the aggregate of all the values above

Curve representing $\frac{1}{24}$ part of the aggregate, supposing the number of curves increased to 24.



of the values above

Curve representing $\frac{1}{24}$ part of the aggregate, supposing the number of curves increased to 24.



J. Basire lith.

tina whose ordinate is b, becomes, in the case in which the eye is too far from the luminous point to see it distinctly,

$$2 - G\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right).\cos\varphi\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right)$$

$$+ G\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right).\cos R.\cos\varphi\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right)$$

$$+ G\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right).\sin R.\sin\varphi\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right)$$

$$= 2 - G\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right).\cos\varphi\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right)$$

$$+ G\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right).\cos\left\{\varphi\left(\sqrt{\frac{2a}{\lambda c e}\cdot\frac{cb}{a}+g}\right)-R\right\}.$$

And similarly, in the case in which the eye is too near to see distinctly, the expression for the intensity is

$$2 - G\left(\sqrt{\frac{2a'}{\lambda c e}\cdot\frac{cb}{a'}-g}\right).\cos\varphi\left(\sqrt{\frac{2a'}{\lambda c e}\cdot\frac{cb}{a'}-g}\right)$$

$$+ G\left(\sqrt{\frac{2a'}{\lambda c e}\cdot\frac{cb}{a'}-g}\right).\cos\left\{\varphi\left(\sqrt{\frac{2a'}{\lambda c e}\cdot\frac{cb}{a'}-g}\right)-R\right\}.$$

It must be remarked that, in every part of the investigation, we suppose the square root to be taken with the positive sign.

To present to the eye a representation of the value of $G(s)\cos\varphi(s)$, upon which the whole of our computations now depend, I have constructed the curve, Plate VIII. fig. 1., in which the abscissa represents the value of s, and the ordinate represents the value of $G(s)\cos\varphi(s)$. The curve corresponding to the value of $G(s)\cos\{\varphi(s)-R\}$ may be sufficiently well inferred from this, by conceiving the whole curve to be pushed on, not through the same space in all parts, but in different parts through different spaces, bearing always the same proportion to the length of one of the waves which R bears to 360°. Thus $G(s)\cos\{\varphi(s)-90^\circ\}$ is represented by the curve in fig. 2.

From an inspection of fig. 1, the following points are easily ascertained. First, that the variations of intensity of light, which are represented by $G(s).\cos\varphi(s)$, are so small when s is large, that they might on that account alone be rejected. Secondly, that if the intensities of a great number of non-interfering streams of light be aggregated, the origin of s for each of the streams having a different position, but the intermediate distances of these origins being small: the variations of intensity near the origins of s may all sensibly coincide so as to produce a set of strong alternations of light and dark in the aggregate; while at the places where s is large, the small distances of the origins and the corresponding displacement of the waves of the curve will be sufficiently great to cause the elevated parts of one curve to answer to the depressed parts of another, &c., or the strong light of one stream to be mingled with

the weak light of another; and thus the alternations of light and dark will be visible only near the origins of s. Thirdly, that if the origins of s in the non-interfering streams of light do coincide, but if the values of λ are very different, the values of s for the same value of b $\left(\text{expressed by } \sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right)$ may, when b is large, differ very much for the different values of λ, and thus the strong light of one stream will be mingled with the weak light of another: and here also the alternation of light and dark will be visible only near the origins of s. I will now proceed with practical applications of our formulæ.

1. Suppose that heterogeneous light issues from a point or a narrow line, and is viewed by an eye too distant to see it distinctly: a thin plate of mica, with its edge parallel to the line, is gradually brought across the pupil of the eye: to describe the appearance of the line.

The formula is

$$2 - G\left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right) . \cos \varphi \left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right)$$
$$+ G\left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right) \cos \left\{\varphi \left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right) - R\right\};$$

and the aggregate of all the values of this expression for the different values of λ is to be taken. Now the last term may be rejected at once. For the values of R will vary very much, perhaps to the extent of many multiples of 360°, for the differently coloured rays; and therefore, in the aggregate, the expressions

$$\cos \left\{\varphi \left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right) - R\right\}$$

which are added together, will have had all values, positive and negative. The expression is therefore

$$2 - G\left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right) \cos \varphi \left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cb}{a} + g}\right).$$

The alternations of light represented by this, for the reasons above mentioned, will be sensible only where $\frac{cb}{a} + g$ is small, that is, where b differs little from $-\frac{ag}{c}$. If then the edge of the mica is on the *right* side of the centre of the pupil, the bands of light will be on the *left* side of the centre of the confused image on the retina, or will give to the mind the perception of bands of light on the *right* side of the object viewed: and therefore, upon moving the plate of mica, the bands will appear to move in the same direction. Moreover, the band which is most conspicuous, because its coefficient of the variable part is largest, because it is unaffected by the variation of λ, and because the other bands are ranged symmetrically on both sides, is that for which $\frac{cb}{a} + g$ is 0, and for this the expression of intensity for every colour is $2 - G(0) \cos \varphi(0) = 2 - 1 = 1$, which is its minimum value. Hence there will be seen a central dark

band strongly marked, with bright and dark bands on each side faintly marked. And the whole of these phenomena will be independent of the thickness of the mica (within wide limits). The reader may very easily verify these conclusions by experiment; and the whole will be found strictly in agreement with observation.

If the eye had been supposed too near to see the line of light distinctly, the investigation would have been precisely the same, but the place where the bands are sensible would have been determined by making $\frac{cb}{a'} - g$ small, or b nearly equal to $+\frac{a'g}{c}$; from which it will appear that, upon moving the plate of mica, the bands will appear to move in the opposite direction; which agrees with observation.

2. Suppose that the linear origins of the various kinds of homogeneous light are separated, either by prism-refraction, or by the diffraction of a grating, or in any other way which arranges the colours in an order corresponding to the order of the values of λ; suppose that the eye is too distant to see the lines of colour distinctly; a thin plate of mica, with its edge parallel to the lines, is gradually brought across the pupil of the eye: to describe the appearance of the spectrum.

First, suppose the red end of the spectrum to be on the same side as the plate of mica, or on that side on which b and g are considered positive.

Let k be the ordinate measured from a fixed point on the retina to the centre of the confused image of any one colour (k therefore is a function of λ); and let l be the ordinate, measured from the same fixed point, of any point of which the intensity of light is to be ascertained. Then $k + b = l$, or $b = l - k$. Substituting this in the general expression for intensity, it becomes

$$2 - \mathrm{G}\left(\sqrt{\frac{2a}{\lambda c e}}\cdot\overline{\frac{cl}{a}+g-\frac{ck}{a}}\right).\cos\varphi\left(\sqrt{\frac{2a}{\lambda c e}}\cdot\overline{\frac{cl}{a}+g-\frac{ck}{a}}\right)$$
$$+\,\mathrm{G}\left(\sqrt{\frac{2a}{\lambda c e}}\cdot\overline{\frac{cl}{a}+g-\frac{ck}{a}}\right).\cos\left\{\varphi\left(\sqrt{\frac{2a}{\lambda c e}}\cdot\overline{\frac{cl}{a}+g-\frac{ck}{a}}\right)-\mathrm{R}\right\}.$$

Now the second term of this expression may at once be neglected, without consideration of the position of the spectrum. For as k is a function of λ, and as the spectral separation is considerable, $\sqrt{\frac{2a}{\lambda c e}}\cdot\frac{ck}{a}$ will vary rapidly with λ, and therefore the angle $\varphi\left(\sqrt{\frac{2a}{\lambda c e}}\cdot\overline{\frac{cl}{a}+g-\frac{ck}{a}}\right)$ may have all the values included in several circumferences, for the variation of λ included in those rays which fall on the same point l; and the positive and negative values of the cosine will sensibly destroy each other.

With regard to the third term, it must be remarked that, in the image formed upon the retina, the blue end of the spectrum is on the same side as the plate of mica, or that k is greatest for the most refrangible rays, and therefore

$$\varphi\left(\sqrt{\frac{2a}{\lambda c e}}\cdot\overline{\frac{cl}{a}+g-\frac{ck}{a}}\right)$$

is least for the most refrangible rays. Moreover, R is greatest, or $-$ R is least, for the most refrangible rays. Hence the effect of the addition of the term $-$ R is to make the variation of the argument of the cosine still more rapid for the variation of λ, and the positive and negative values of the cosine will therefore destroy each other, or the third term may be neglected.

The expression for intensity is, therefore, reduced to its first term 2, or there are no visible bands in the spectrum.

Secondly, suppose the blue end of the spectrum to be on the same side as the plate of mica.

The second term of the expression may be neglected, as before. But with regard to the third term, the circumstances are entirely different. For k is now least for the most refrangible rays (the blue end of the spectrum formed on the retina being on the side opposite to the mica), and therefore $\phi\left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cl}{a} + g - \frac{ck}{a}}\right)$ is greatest for the most refrangible rays; and therefore the chromatic variations of the different parts of the argument $\phi\left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cl}{a} + g - \frac{ck}{a}}\right) - \mathrm{R}$ have a tendency to destroy each other. And by proper selection of the thickness of the piece of mica, the chromatic variations of R (for the colours which fall upon the same point of the retina) may be the same as the chromatic variations of $\phi\left(\sqrt{\frac{2a}{\lambda c e}} \cdot \overline{\frac{cl}{a} + g - \frac{ck}{a}}\right)$, for those values of the function which make the bands most brilliant. In this case, then, the bands produced by all the neighbouring colours will be aggregate in intensity, and therefore strong bands will be seen on the spectrum.

With regard to the place at which any bright or dark band is seen, as depending on the place of the edge of the mica; that is, with regard to the value of l for one of these bands, as depending on g; it will be evident that the intensity (whether strongest or weakest) will be preserved the same by keeping $\frac{cl}{a} + g$ the same; that is, there will be a band of the same character so long as l varies in the same degree as $\frac{ag}{c}$, but in the opposite direction; that is, the band upon the retina will shift in the direction opposite to the shift of the mica, or will appear to the mind to shift in the same direction as the mica. But this shift will be small when a is small.

3. Suppose that the eye is too near to see the lines of colour distinctly, and that other circumstances are the same as in the second problem.

The expression for intensity in this case is

$$2 - \mathrm{G}\left(\sqrt{\frac{2a'}{\lambda c e}} \cdot \overline{\frac{cl}{a'} - g - \frac{ck}{a'}}\right) . \cos\phi\left(\sqrt{\frac{2a'}{\lambda c e}} \cdot \overline{\frac{cl}{a'} - g - \frac{ck}{a'}}\right)$$

$$+ \mathrm{G}\left(\sqrt{\frac{2a'}{\lambda c e}} \cdot \overline{\frac{cl}{a'} - g - \frac{ck}{a'}}\right) . \cos\left\{\phi\left(\sqrt{\frac{2a'}{\lambda c e}} \cdot \overline{\frac{cl}{a'} - g - \frac{ck}{a'}}\right) - \mathrm{R}\right\}.$$

The investigation of the effect of the different terms may be conducted in the very same words as in the second problem, and the result is precisely the same; namely, that when the red end of the spectrum is on the same side as the mica, no bands are produced; but that when the blue end of the spectrum is on the same side as the mica, and the thickness of the mica is properly adjusted, the bands produced by the neighbouring colours will correspond, or nearly so, and strong bands will therefore be produced in the aggregate effect on the eye.

With regard to the place at which any bright or dark band is seen, as depending on the place of the edge of the mica, the intensity will now be preserved the same by keeping $\frac{c\,l}{a'} - g$ the same; that is, a band of the same character will be preserved by making the variations of l equal to those of $\frac{a'g}{c}$, and in the same direction; or the bands upon the retina will shift in the same direction as the shift of the mica, and the mind will therefore perceive bands to shift in the opposite direction to the shift of the mica.

To exhibit more distinctly to the eye the cause of the annihilation of the bands when the red end of the external spectrum or the blue end of the spectrum on the retina is on the same side as the mica, and the cause of the formation of the bands when the blue end of the external spectrum or the red end of the spectrum on the retina is on the same side as the mica, I have constructed diagrams (figs. 3, 4, 5, 6, 7,) founded on the following calculations. The Table below contains computed values of $2 - G(s) \,.\, \cos\phi(s) + G(s) \,.\, \cos\{\phi(s) - R\}$ for all the values of s which are likely to produce sensible effects in the result; and for the values of R, 0, 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°. These of course apply also to the values of R, $2n\pi$, $2n\pi + 30°$, $2n\pi + 60°$, &c.

Table of the values of $2 - G(s) . \cos \phi(s) + G(s) . \cos \{\phi(s) - R\}$.

s	Values of R.											
	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
−4·2	2·000	2·050	2·146	2·263	2·368	2·433	2·442	2·392	2·296	2·179	2·074	2·009
−4·1	2·000	1·917	1·885	1·914	1·995	2·108	2·222	2·305	2·337	2·308	2·227	2·114
−4·0	2·000	1·902	1·789	1·693	1·639	1·642	1·700	1·798	1·911	2·007	2·061	2·058
−3·9	2·000	2·027	1·995	1·914	1·804	1·696	1·617	1·591	1·623	1·704	1·814	1·922
−3·8	2·000	2·122	2·222	2·273	2·263	2·193	2·084	1·962	1·862	1·811	1·821	1·891
−3·7	2·000	2·066	2·181	2·313	2·428	2·496	2·498	2·432	2·317	2·185	2·070	2·002
−3·6	2·000	1·927	1·921	1·984	2·098	2·233	2·352	2·425	2·431	2·368	2·254	2·119
−3·5	2·000	1·870	1·752	1·676	1·663	1·718	1·824	1·954	2·072	2·148	2·161	2·106
−3·4	2·000	1·960	1·862	1·735	1·613	1·530	1·508	1·548	1·646	1·773	1·895	1·978
−3·3	2·000	2·097	2·132	2·097	2·000	1·868	1·738	1·641	1·606	1·641	1·738	1·870
−3·2	2·000	2·145	2·288	2·391	2·427	2·386	2·378	2·133	1·990	1·887	1·851	1·892
−3·1	2·000	2·060	2·187	2·347	2·496	2·595	2·616	2·556	2·429	2·269	2·120	2·021
−3·0	2·000	1·921	1·923	2·006	2·149	2·313	2·454	2·533	2·531	2·448	2·305	2·141
−2·9	2·000	1·843	1·720	1·664	1·689	1·790	1·938	2·095	2·218	2·274	2·249	2·148
−2·8	2·000	1·890	1·739	1·590	1·482	1·444	1·488	1·598	1·749	1·898	2·006	2·044
−2·7	2·000	2·026	1·966	1·840	1·682	1·534	1·438	1·412	1·472	1·598	1·756	1·904
−2·6	2·000	2·149	2·233	2·230	2·141	1·989	1·816	1·667	1·583	1·586	1·675	1·827
−2·5	2·000	2·186	2·372	2·508	2·558	2·508	2·372	2·186	2·000	1·864	1·814	1·864
−2·4	2·000	2·117	2·303	2·512	2·687	2·781	2·770	2·653	2·467	2·258	2·083	1·989
−2·3	2·000	1·980	2·071	2·249	2·468	2·668	2·796	2·816	2·725	2·547	2·328	2·128
−2·2	2·000	1·848	1·797	1·861	2·023	2·238	2·450	2·602	2·653	2·589	2·427	2·212
−2·1	2·000	1·786	1·618	1·540	1·574	1·711	1·914	2·128	2·296	2·374	2·340	2·203
−2·0	2·000	1·821	1·601	1·412	1·304	1·304	1·426	1·605	1·825	2·014	2·122	2·122
−1·9	2·000	1·925	1·759	1·548	1·348	1·213	1·181	1·257	1·423	1·634	1·834	1·969
−1·8	2·000	2·071	2·020	1·866	1·647	1·425	1·260	1·189	1·240	1·394	1·613	1·835
−1·7	2·000	2·203	2·300	2·267	2·112	1·878	1·628	1·425	1·328	1·361	1·516	1·750
−1·6	2·000	2·284	2·514	2·639	2·599	2·430	2·168	1·884	1·654	1·529	1·569	1·738
−1·5	2·000	2·301	2·619	2·871	2·989	2·941	2·742	2·441	2·123	1·871	1·753	1·801
−1·4	2·000	2·252	2·598	2·948	3·206	3·304	3·318	2·966	2·620	2·270	2·012	1·914
−1·3	2·000	2·150	2·461	2·851	3·217	3·460	3·516	3·366	3·055	2·665	2·299	2·056
−1·2	2·000	2·015	2·240	2·615	3·041	3·402	3·604	3·589	3·364	2·989	2·563	2·202
−1·1	2·000	1·874	1·979	2·288	2·719	3·157	3·486	3·612	3·507	3·198	2·767	2·329
−1·0	2·000	1·740	1·709	1·918	2·309	2·778	3·202	3·462	3·493	3·284	2·893	2·424
−0·9	2·000	1·626	1·461	1·548	1·863	2·323	2·802	3·176	3·341	3·254	2·939	2·479
−0·8	2·000	1·548	1·263	1·221	1·433	1·842	2·338	2·790	3·075	3·117	2·905	2·496
−0·7	2·000	1·496	1·113	0·946	1·042	1·373	1·852	2·350	2·733	2·900	2·804	2·473
−0·6	2·000	1·490	1·036	0·759	0·734	0·969	1·400	1·910	2·364	2·641	2·666	2·431
−0·5	2·000	1·504	1·006	0·639	0·500	0·628	0·986	1·482	1·980	2·347	2·486	2·358
−0·4	2·000	1·545	1·029	0·590	0·347	0·364	0·636	1·091	1·607	2·046	2·289	2·272
−0·3	2·000	1·604	1·095	0·609	0·275	0·184	0·360	0·756	1·265	1·751	2·085	2·176
−0·2	2·000	1·681	1·200	0·688	0·281	0·087	0·160	0·479	0·960	1·472	1·879	2·073
−0·1	2·000	1·769	1·338	0·821	0·358	0·072	0·040	0·271	0·702	1·219	1·682	1·968
0·0	2·000	1·866	1·500	1·000	0·500	0·134	0·000	0·134	0·500	1·000	1·500	1·866
+0·1	2·000	1·968	1·682	1·219	0·702	0·271	0·040	0·072	0·358	0·821	1·338	1·769
+0·2	2·000	2·073	1·879	1·472	0·960	0·479	0·160	0·087	0·281	0·688	1·200	1·681
+0·3	2·000	2·176	2·085	1·751	1·265	0·756	0·360	0·184	0·275	0·609	1·095	1·604
+0·4	2·000	2·272	2·289	2·046	1·607	1·091	0·636	0·364	0·347	0·590	1·029	1·545
+0·5	2·000	2·358	2·486	2·347	1·980	1·482	0·986	0·628	0·500	0·639	1·006	1·504
+0·6	2·000	2·431	2·666	2·641	2·364	1·910	1·400	0·969	0·734	0·759	1·036	1·490
+0·7	2·000	2·473	2·804	2·900	2·733	2·350	1·852	1·373	1·042	0·946	1·113	1·496
+0·8	2·000	2·496	2·905	3·117	3·075	2·790	2·338	1·842	1·433	1·221	1·263	1·548
+0·9	2·000	2·479	2·939	3·254	3·341	3·176	2·802	2·323	1·863	1·548	1·461	1·626
+1·0	2·000	2·424	2·893	3·284	3·493	3·462	3·202	2·778	2·309	1·918	1·709	1·740
+1·1	2·000	2·329	2·767	3·198	3·507	3·612	3·486	3·157	2·719	2·288	1·979	1·874
+1·2	2·000	2·202	2·563	2·989	3·364	3·589	3·604	3·402	3·041	2·615	2·240	2·015
+1·3	2·000	2·056	2·299	2·665	3·055	3·366	3·516	3·460	3·217	2·851	2·461	2·150

TABLE. (Continued.)

s	Values of R.											
	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
+1·4	2·000	1·914	2·012	2·270	2·620	2·966	3·318	3·304	3·206	2·948	2·598	2·252
+1·5	2·000	1·801	1·753	1·871	2·123	2·441	2·742	2·941	2·989	2·871	2·619	2·301
+1·6	2·000	1·738	1·569	1·529	1·654	1·884	2·168	2·430	2·599	2·639	2·514	2·284
+1·7	2·000	1·750	1·516	1·361	1·328	1·425	1·628	1·878	2·112	2·267	2·300	2·203
+1·8	2·000	1·835	1·613	1·394	1·240	1·189	1·260	1·425	1·647	1·866	2·020	2·071
+1·9	2·000	1·969	1·834	1·634	1·423	1·257	1·181	1·213	1·348	1·548	1·759	1·925
+2·0	2·000	2·122	2·122	2·014	1·825	1·605	1·426	1·304	1·304	1·412	1·601	1·821
+2·1	2·000	2·203	2·340	2·374	2·296	2·128	1·914	1·711	1·574	1·540	1·618	1·786
+2·2	2·000	2·212	2·427	2·589	2·653	2·602	2·450	2·238	2·023	1·861	1·797	1·848
+2·3	2·000	2·128	2·328	2·547	2·725	2·816	2·796	2·668	2·468	2·249	2·071	1·980
+2·4	2·000	1·989	2·083	2·258	2·467	2·653	2·770	2·781	2·687	2·512	2·303	2·117
+2·5	2·000	1·864	1·814	1·864	2·000	2·186	2·372	2·508	2·558	2·508	2·372	2·186
+2·6	2·000	1·827	1·675	1·586	1·583	1·667	1·816	1·989	2·141	2·230	2·233	2·149
+2·7	2·000	1·904	1·756	1·598	1·472	1·412	1·438	1·534	1·682	1·840	1·966	2·026
+2·8	2·000	2·044	2·006	1·898	1·749	1·598	1·488	1·444	1·482	1·590	1·739	1·890
+2·9	2·000	2·148	2·249	2·274	2·218	2·095	1·938	1·790	1·689	1·664	1·720	1·843
+3·0	2·000	2·141	2·305	2·448	2·531	2·533	2·454	2·313	2·149	2·006	1·923	1·921
+3·1	2·000	2·021	2·120	2·269	2·429	2·556	2·616	2·595	2·496	2·347	2·187	2·060
+3·2	2·000	1·892	1·851	1·887	1·990	2·133	2·378	2·386	2·427	2·391	2·288	2·145
+3·3	2·000	1·870	1·738	1·641	1·606	1·641	1·738	1·868	2·000	2·097	2·132	2·097
+3·4	2·000	1·978	1·895	1·773	1·646	1·548	1·508	1·530	1·613	1·735	1·862	1·960
+3·5	2·000	2·106	2·161	2·148	2·072	1·954	1·824	1·718	1·663	1·676	1·752	1·870
+3·6	2·000	2·119	2·254	2·368	2·431	2·425	2·352	2·233	2·098	1·984	1·921	1·927
+3·7	2·000	2·002	2·070	2·185	2·317	2·432	2·498	2·496	2·428	2·313	2·181	2·066
+3·8	2·000	1·891	1·821	1·811	1·862	1·962	2·084	2·193	2·263	2·273	2·222	2·122
+3·9	2·000	1·922	1·814	1·704	1·623	1·591	1·617	1·696	1·804	1·914	1·995	2·027
+4·0	2·000	2·058	2·061	2·007	1·911	1·798	1·700	1·642	1·639	1·693	1·789	1·902
+4·1	2·000	2·114	2·227	2·308	2·337	2·305	2·222	2·108	1·995	1·914	1·885	1·917
+4·2	2·000	2·009	2·074	2·179	2·296	2·392	2·442	2·433	2·368	2·263	2·146	2·050

Fig. 3. contains a graphical representation of the variations of intensity corresponding to these twelve different values of R, the elevation of the curve above its straight line representing the intensity, as given in the Table; the curve at the bottom represents the aggregate in the same manner (the sums of the ordinates being divided by twelve): this figure corresponds to the case of the first problem, excepting that, as no variation is made in the value of λ for the different curves, $\left(\text{which ought to be done, because } s \text{ here} = \sqrt{\frac{2a}{\lambda c e} \cdot \frac{c b}{a} + g},\right)$ the destruction of the bands in receding from the centre is not properly represented; the central bands however are properly represented. Fig. 4. contains a representation of the effect of shifting the central origin of each set of bands by a space proportionate to R, so that the shift 1·8 in the values of s corresponds to 360° of R, the direction of the shift being that produced by placing the red end of the spectrum next to the mica; the first curve at the bottom represents the aggregate in the same manner (the sum of the ordinates being divided by twelve); the second curve represents the aggregate, supposing twenty-four curves taken, the shift of s in the twenty-fourth being double of that in the twelfth; the de-

struction of bands is here, as to sense, complete, although the shift of s is the same in amount as that which (when taken the opposite way) produces strong bands. Fig. 5. contains a representation of the effect of shifting the bands in the direction of the shift produced by placing the blue end of the spectrum next to the mica, so that the shift 1·2 in the value of s corresponds to 360° of R. Fig. 6. has the same with the shift 1·8 in the value of s corresponding to 360° of R; and fig. 7. has the same with the shift 2·4 in the value of s corresponding to 360° of R. Each of these figures has at the bottom a curve showing the effect of the aggregation, supposing twelve curves taken (the sum of the ordinates being divided by twelve), and another showing the effect supposing twenty-four curves taken (the sum of the ordinates being divided by twenty-four), the value of R for the twenty-fourth being double of that for the twelfth. The intensity of the bands in the result is here seen well.

It must be remarked that no sensible error is produced here by making no variation in the value of λ. For the different kinds of light which are mingled are not those from all parts of the spectrum, but those from parts of the spectrum of a very limited extent; and the properties of the difference of quality of the different rays enter, not immediately in consequence of the variation of λ, but in consequence of the variation of R.

It is also to be remarked that strong bands are produced as well in fig. 5. as in fig. 7, although in the former there is a shift of 1·2 in the value of s corresponding to 360° of R, and in the latter a shift of 2·4 in the value of s corresponding to 360° of R. It appears therefore that, with the same spectrum, considerable latitude in the thickness of the mica is allowable, or considerable latitude in the degree to which the spectrum is viewed out of focus. For the same reason, the same plate of mica, which exhibits bands in the spectrum formed by a prism, may also exhibit bands in the spectrum formed by a grating upon a lens or by reflection from a striated surface, though the proportionate degree of separation of the colours, in different parts of the spectrum, is exceedingly different in these cases.

The intervals of the bands will, however, always be approximately determined by certain numerical changes in the value of R, and there will, therefore, always be *nearly* the same number of bands on the spectrum, and this number will always be *nearly* the same as the number of bands remarked by Mr. TALBOT when the spectrum is seen distinctly. The bands, therefore, will generally become broader as the spectrum becomes broader, that is, as the eye (supposed at first too far off for distinct vision) approaches to the position of distinct vision. It may, however, happen that, as a becomes small, the changes in the value of s will not nearly correspond to those of R, and therefore, in a position intermediate to that at which these bands and that at which Mr. TALBOT's bands are clearly seen, no bands whatever may be visible.

In the whole of the special conclusions deduced from the theory, the agreement with observation is complete. I wish it to be clearly understood that I confine this statement to the general appearance of the phenomena, for measures are yet

wanting in every part; and some of them, depending, as they must, on the variable state of the eye as to focal adjustment during the observation, cannot be obtained with ease or certainty.

The train of phenomena and their related theory may be considered remarkable: in the first place for the apparent obscurity of the explanation, which was sufficient to induce an experienced philosopher to ascribe the appearances to a new property of light; in the next place for the unexpected simplicity of the relation between the numbers which occur in the investigation, which relation contributes materially to facilitate the understanding of the results deduced from them; and lastly, I may perhaps add, for the completeness of the explanation which the phenomenon receives from the undulatory theory.

Royal Observatory, Greenwich,
May 30, 1840.

Numerical values of the ordinates of the curves in figures 4, 5, 6, and 7, representing $\frac{1}{12}$th part of the aggregate of the values of $2-G(s).\cos\phi(s)+G(s).\cos\{\phi(s)-R\}$ *for 12 values of R; and also, of the curves representing $\frac{1}{24}$th part of the aggregate of the values for 24 values of R: for values of s increasing 0·1 at each step.*

For Figure 4.				For Figure 5.				For Figure 6.				For Figure 7.			
One twelfth part for 12 values of R.	One twelfth part for 12 values of R. Continued.	One twenty-fourth part for 24 values of R.	One twenty-fourth part for 24 values of R. Continued.	One twelfth part for 12 values of R.	One twelfth part for 12 values of R. Continued.	One twenty-fourth part for 24 values of R.	One twenty-fourth part for 24 values of R. Continued.	One twelfth part for 12 values of R.	One twelfth part for 12 values of R. Continued.	One twenty-fourth part for 24 values of R.	One twenty-fourth part for 24 values of R. Continued.	One twelfth part for 12 values of R.	One twelfth part for 12 values of R. Continued.	One twenty-fourth part for 24 values of R.	One twenty-fourth part for 24 values of R. Continued.
2·139	2·139	2·077	1·915	1·987	1·170	2·109	1·717	2·018	0·784	2·207	1·434	2·046	0·983	1·891	1·502
2·123	2·142	2·071	1·906	1·996	0·987	2·115	1·766	2·016	0·755	2·293	1·369	2·057	1·121	1·785	1·582
2·108	2·131	2·069	1·898	2·020	0·855	2·073	1·816	2·006	0·784	2·346	1·356	2·057	1·303	1·680	1·680
2·069	2·112	2·050	1·892	2·043	0·776	1·994	1·851	1·997	0·872	2·381	1·393	2·042	1·513	1·582	1·785
2·039	2·091	2·038	1·891	2·066	0·749	1·914	1·858	2·014	1·016	2·384	1·471	2·022	1·737	1·502	1·891
2·010	2·071	2·023	1·896	2·054	0·776	1·853	1·831	2·048	1·210	2·352	1·583	2·002	1·956	1·448	1·992
1·982	2·053	2·004	1·906	2·024	0·855	1·850	1·776	2·081	1·447	2·282	1·723	1·983	2·155	1·425	2·093
1·954	2·038	1·983	1·920	1·974	0·987	1·906	1·699	2·095	1·711	2·174	1·878	1·971		1·433	
1·922	2·024	1·964	1·935	1·936	1·170	2·020	1·637	2·075	1·987	2·031	2·034	1·968		1·475	
1·886	2·012	1·949	1·949	1·948	1·400	2·179	1·619	2·045	2·253	1·878	2·174	1·987		1·554	
1·845	2·007	1·935	1·964	2·026	1·669	2·362	1·672	2·000	2·484	1·723	2·282	2·028		1·666	
1·802	2·013	1·920	1·983	2·142	1·964	2·514	1·809	1·957	2·657	1·583	2·352	2·095		1·804	
1·760	2·025	1·907	2·004	2·231	2·265	2·599	2·013	1·926	2·754	1·471	2·384	2·175		1·956	
1·721	2·036	1·896	2·023	2·234	2·546	2·580	2·245	1·913	2·766	1·393	2·381	2·264		2·110	
1·691	2·038	1·891	2·038	2·127	2·777	2·452	2·452	1·921	2·695	1·356	2·346	2·347		2·251	
1·671	2·031	1·892	2·050	1·944	2·926	2·245	2·580	1·984	2·571	1·369	2·293	2·421		2·369	
1·664	2·031	1·898	2·069	1·761	2·966	2·013	2·599	2·083	2·396	1·434	2·207	2·488		2·462	
1·671	2·018	1·906	2·071	1·652	2·886	1·808	2·514	2·227	2·227	1·549	2·127	2·519		2·512	
1·691		1·915		1·676	2·697	1·672	2·362	2·396		1·706		2·530		2·530	
1·722		1·922		1·838	2·411	1·619	2·182	2·571		1·890		2·505		2·512	
1·760		1·934		2·104	2·104	1·637	2·020	2·695		2·071		2·470		2·462	
1·802		1·936		2·411	1·838	1·699	1·906	2·766		2·239		2·317		2·369	
1·846		1·942		2·698	1·676	1·776	1·850	2·754		2·371		2·155		2·251	
1·886		1·948		2·886	1·652	1·831	1·853	2·657		2·455		1·956		2·110	
1·922		1·952		2·966	1·761	1·858	1·913	2·484		2·484		1·737		1·956	
1·954		1·954		2·926	1·944	1·851	1·994	2·253		2·455		1·513		1·804	
1·982		1·952		2·777	2·127	1·816	2·073	1·987		2·371		1·303		1·666	
2·010		1·948		2·546	2·233	1·766	2·115	1·711		2·239		1·121		1·554	
2·039		1·942		2·264	2·231	1·717	2·109	1·447		2·071		0·983		1·475	
2·069		1·936		1·964	2·142	1·682	2·072	1·210		1·890		0·894		1·433	
2·107		1·934		1·669		1·669		1·016		1·706		0·866		1·425	
2·123		1·922		1·400		1·682		0·872		1·549		0·894		1·447	

Figure 1

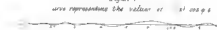


Figure 2

Figure 3

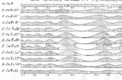


Figure 4

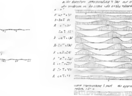


Figure 5

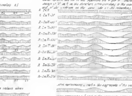


Figure 6

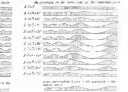


Figure 7